PLASMA MODES ALONG THE OPEN FIELD LINES OF A NEUTRON STAR

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ABSTRACT

We consider electrostatic plasma modes along the open field lines of a rotating neutron star. Goldreich-Julian charge density in general relativity is analyzed for the neutron star with zero inclination. It is found that the charge density is maximum at the polar cap and it remains almost same in certain extended region of the pole. For a steady state Goldreich-Julian charge density we found the usual plasma oscillation along the field lines; plasma frequency resembles to the gravitational redshift close to the Schwarzschild radius. We study the nonlinear plasma mode along the field lines. From the system of equations under general relativity, a second order differential equation is derived. The equation contains a term which describes the growing plasma modes near Schwarzschild radius in a black hole environment. The term vanishes with the distance far away from the gravitating object. For initially zero potential and field on the surface of a neutron star, Goldreich-Julian charge density is found to create the plasma mode, which is enhanced and propagates almost without damping along the open field lines. We briefly outline our plan to extend the work for studying soliton propagation along the open field lines of strongly gravitating objects.

Subject headings: MHD - plasmas - pulsars: plasma: general - relativity - stars: neutron

1. INTRODUCTION

Study of plasma modes in the neutron star or black hole environments is related with the investigation of radio emissions coming from these sources (see, e.g. Buzzi et al. 1995, Mofiz 1997 and the references therein). Radio pulsars which are rotating neutron stars with spin periods ranging from ms to 5 s, are characterized by surface magnetic fields of the order of 10^{12} G, radii of about 10 km and central densities in excess of $10^{14}\ g \cdot cm^3$, and so are purely gravitating objects. A spining magnetized neutron star generates huge potential differences between different parts of its surface (Goldreich & Julian 1969). The cascade generation of electron-positron plasmas in the polar cap region (Sturrock 1971, Ruderman & Sutherland 1975) means that the magnetosphere of a neutron star is filled with plasma - screening the longitudinal electric field. This screening results in the corotation of plasma with a star. Such a rotation is not possible outside the light cylinder, thus it forms essentially different groups of field lines: closed i. e. those returning the stellar surface, and open, i.e. those crossing the light cylinder and going to infinity. As a result, plasma may leave the neutron star along the open field lines. The charges along the field lines create plasma modes which may be related with the pulsar radiation and with its microstructures.

Our study of plasma modes along the field lines is boosted by the pioneering works of Goldreich & Julian (1969), Sturrock (1971), Mestel (1971), Ruderman & Sutherland (1975) and Arons & Scharleman (1979). The subsequent achievements and some new ideas are reviewed by Arons (1991), Michel (1991), Mestel (1992) and Muslimov & Harding (1997). Although a self consistent pulsar magnetosphere theory is yet to developed, the analysis of plasma modes in the pulsar magnetosphere based on the above mentioned works provides firm grounds for the construction of such a model.

In this paper, we attempt to extend Muslimov & Harding work (1997) to study plasma modes along the open field lines of a rotating neutron star. In §2 general relativistic equations describing the electrodynamics of a rotating neutron star are formulated. The equations are rewritten in the frame of reference corotating with the neutron star. We deduce the general system of equations governing the electrostatic modes in the pulsar magnetosphere. A detail analysis of

Goldreich-Julian charge density in general relativity in done in §3. It is shown that the charge density exponentially decays with the distance away from the surface of the star while it has a periodical dependence on the polar angle along the surface. The field is maximum at the polar cap region and it remains almost same in certain extended region in the pole. In §4 we study the linear plasma modes along the open field lines. A general equation governing electrostatic potential is derived. For a steady state Goldreich-Julian charge density, the usual plasma oscillation along the field lines is found. Plasma frequency resembles to the gravitational redshift close to Schwarzschild radius while at a large distance from the gravitational radius, it is the usual plasma oscillation along the field lines. In §5 we study the nonlinear plasma modes along the field lines. From the system equations under general relativity, a second order differential equation is derived. The equation contains a term which describes the growing plasma mode near the Schwarzschild radius of a neutron star or a black hole. The term vanishes with the distance far away of the gravitating object. The equation is solved numerically subjected to appropriate boundary conditions. It is found that Goldreich-Julian charge density creates the initial field on the surface of the star which is enhanced near the gravitational radius and almost without damping propagates along the open field lines. In §6 we conclude our findings and discuss them for further investigations.

2. GENERAL RELATIVISTIC ELECTRODYNAMIC EQUATIONS IN THE COROTATING FRAME OF REFERENCE

Recently Muslimov and Harding (1997) derived the general relativistic electrodynamic equations for a neutron star in the corotating frame of reference. It is noted that the effects of general relativity are very important : the dragging of inertial frames of reference significantly affects the electric field generated in the vicinity of a rotating magnetized neutron star, while the static part of the gravitational field results in additional enhancement of electric and magnetic fields near a star.

The metric of an asymptotically flat, stationary, axially symmetric spacetime around a rotating gravitating body (see, e.g. Landau & Lifshitz 1975) is considered. In spherical polar

coordinates $x^0 = ct, x^1 = r, x^2 = \theta$ and $x^3 = \phi$, we have

$$ds^2 = A^2(cdt)^2 - B^2(dr)^2 - C^2(d\theta)^2 - D^2(d\phi - \omega dt)^2, \tag{1}$$

where $A = B^{-1} = (1 - r_g/r)^{1/2}$ is the gravitational redshift function, $C = r, D = r\sin\theta$, $r_g = 2GM/c^2$ is the gravitational radius of body (neutron star) of mass M, J is the angular momentum of a neutron star, c is the speed of light and G is the gravitational constant. The metric in equation (1) is the approximation of Kerr metric when the ratio J/Mcr_g is small. The presence of the nondiagonal component in metric in equation (1) results in the well known effect of dragging of inertial frames of reference (the Lense-Thirring effect) with the angular velocity

$$\omega = \frac{2GJ}{c^2 r^3}. \tag{2}$$

The metric in equation (1) can be transformed to the frame of reference corotating with a neutron star:

$$ds^2 = A^2(cdt)^2 - B^2(dr)^2 - c^2(d\theta)^2 - D^2(d\phi - \Omega dt)^2, \tag{3}$$

by transformations $t' = t, r^{'} = r, \theta^{'} = \theta, \varphi^{'} = \varphi - \Omega_0 t$. Here $\Omega = \omega - \Omega_0$, where Ω_0 is the angular velocity of rotation of star relative to the distant observer.

The zero angular momentum observer (ZAMO; see, e.g. Thorne, Price & Macdonald 1986) has the four-velocity

$$e^{\nu}\{\frac{1}{\sqrt{1 - r_g/r}}, 0, 0, -\frac{\Omega}{c\sqrt{1 - r_g/r}}\}; \qquad e_{\nu}\{-\sqrt{1 - r_g/r}, 0, 0, 0\} \ . \tag{4}$$

Then the general-relativistic Maxwell equations for observer (eq.[4]) in the metric (eq.[3]) take the form

$$\nabla \cdot \mathbf{B} = \mathbf{0} \ , \tag{5a}$$

$$\nabla \times (\alpha\mathbf{E} - (\omega - \mathbf{\Omega_0}) \times \mathbf{B}) = -\frac{\mathbf{1}}{\mathbf{c}}\frac{\partial \mathbf{B}}{\partial \mathbf{t}}, \tag{5b}$$

$$\nabla \cdot \mathbf{E} = \mathbf{4}\pi\rho, \tag{5c}$$

$$\nabla \times (\alpha \mathbf{B}) + \nabla \times ((\omega - \mathbf{\Omega_0}) \times \mathbf{E}) - (\omega - \mathbf{\Omega_0})\,(\nabla \cdot \mathbf{E}) = \frac{\mathbf{1}}{\mathbf{c}}\frac{\partial \mathbf{E}}{\partial \mathbf{t}} + \frac{\mathbf{4}\pi}{\mathbf{c}}\alpha \mathbf{j}\ , \tag{5d}$$

where $(\mathbf{\Omega} \times \mathbf{B})^\alpha = e^{\alpha\beta\mu\nu} e_\beta \Omega_\mu B_\nu$, $(\omega - \mathbf{\Omega_0})_\alpha = \left\{0, 0, 0, \left[(\omega - \Omega_0)\, r \sin\theta\right]/c\sqrt{1 - r_g/r}\right\}$.

Similarly, we may write the charge continuity equation in the above mentioned frame as

$$\frac{\partial \rho}{\partial t} + \left(\frac{\kappa}{r^3} - 1\right)\Omega_0 \mathbf{m} \cdot \nabla\rho + \nabla \cdot (\alpha \mathbf{j}) = \mathbf{0}. \tag{6}$$

Finally, the equation of motion of a charged particle is

$$\frac{1}{\alpha}\frac{d\mathbf{p}}{dt} = m\gamma \mathbf{g} + q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right) + \mathbf{f}\ . \tag{7}$$

Here $\alpha = (1 - r_g/r)^{1/2}$ ($\equiv A$, as denoted in the metric in equation (1)), the parameter $\kappa \equiv 2r_g R^2/5$, R is the radius a neutron star, $\rho = \sum_s n_s q_s, \mathbf{j} = \sum_s n_s q_s \mathbf{v_s}$, $\mathbf{v_s}$ is the velocity, q_s is the charge of particle, n_s is the particle number density, and summation is over all the species; $\mathbf{p} = m\gamma\mathbf{v}$ is the momentum of the particle, $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor, m is the rest mass of the particle, $\mathbf{f}$ is an external force other than electromagnetic, and $\mathbf{g}$ is the gravitational acceleration. All electrodynamic quantities as magnetic field $\mathbf{B}$, electric field $\mathbf{E}$, conduction current $\mathbf{j}$, and charge density ρ in these equations are such as measured by ZAMO (eq.[4]). Gradient, curl and divergence are taken along the curvilinear coordinate

$$e_{\hat{r}} = Ae_r = A\frac{\partial}{\partial r}\ , \quad e_{\hat{\theta}} = \frac{1}{r}e_\theta = \frac{1}{r}\frac{\partial}{\partial\theta}\ , \quad e_{\hat{\phi}} = \frac{1}{r\sin\theta}e_\phi = \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\ , \tag{1}$$

$\mathbf{m} = \mathbf{r}\sin\theta\mathbf{e}_\phi$ is the Killing vector, responsible for axial symmetry.

Assuming $\frac{\partial \mathbf{B}}{\partial t} = 0$ in equation (5b) (i.e. considering magnetic field of a neutron star is stationary in the corotating frame), from equation (5b) we get

$$\alpha\mathbf{E} - (\omega - \mathbf{\Omega_0}) \times \mathbf{B} = -\nabla\mathbf{\Phi}, \tag{8}$$

where Φ is a scalar electrostatic potential.

Taking the divergence of equation (8) and making use of equation (5c), we get

$$\nabla \cdot \left(\frac{\mathbf{1}}{\alpha} \nabla \mathbf{\Phi} + \frac{\mathbf{1}}{\alpha} (\mathbf{\Omega_0} - \omega) \times \mathbf{B} \right) = -\mathbf{4}\pi\rho \ . \tag{9}$$

Equation (9) can be written as (see Muslimov & Tsygan 1986; Beskin 1990)

$$\nabla \cdot (\frac{\mathbf{1}}{\alpha} \nabla \mathbf{\Phi}) = -\mathbf{4}\pi(\rho - \rho_{\mathbf{GJ}}) \tag{10}$$

where

$$\rho_{GJ} = -\frac{1}{4\pi} \nabla \cdot \left(\frac{\mathbf{1}}{\alpha} (\mathbf{\Omega_0} - \omega) \times \mathbf{B} \right) = -\frac{\mathbf{1}}{\mathbf{4}\pi} \nabla \cdot \left(\frac{\mathbf{1}}{\alpha} (\mathbf{1} - \frac{\kappa}{\mathbf{r^3}}) \mathbf{\Omega_0} \times \mathbf{B} \right) \tag{11}$$

is the relativistic analog of the Goldreich-Julian (1969) charge density.

Finally, the equation of motion of the charged particle is

$$\left[\frac{\partial}{\partial t} + \left(\frac{\kappa}{r^3} - 1 \right) \mathbf{\Omega_0} \cdot \nabla + \alpha \mathbf{v} \cdot \nabla \right] \mathbf{p} = -q \nabla \Phi, \tag{12}$$

where the gravitational acceleration $\mathbf{g}$ and the nonelectromagnetic force $\mathbf{f}$ are justifiably ignored.

3. GOLDREICH-JULIAN CHARGE DENSITY IN GENERAL RELATIVITY

In a pioneering work, Goldreich & Julian (1969) have shown that a strongly magnetized, highly conducting neutron star, rotating about the magnetic axis, would spontaneously build up a charged magnetosphere. The essence of the argument is that it imposes a charge magnetosphere which are subject to enormous unbalanced electric forces parallel to the magnetic field. Goldreich and Julian hypothesized that a far better approximation for the magnetosphere would be shorting-out of the component of $\mathbf{E}$ along $\mathbf{B}$ by charges originating in the star. The magnetospheric charges that maintain $\mathbf{E} \cdot \mathbf{B} = 0$ are themselves subject to the $\mathbf{E} \times \mathbf{B}$ drift which sets them into corotation with the star. Here, we analyze Goldreich-Julian charge density in general relativity.

Assuming zero inclination of the rotating star with the magnetic axis we consider $\mathbf{B} = \{B_r, B_\theta, 0\}$. The components B_r, B_θ in this case were first derived by Ginzburg & Ozernoy

(1964). Later on, similar expressions were derived in a number of papers (see, e.g. Wasserman & Shapiro 1983; Muslimov& Tsygan 1986):

$$B_r = \frac{2\cos\theta}{r^3} f(r)\mu, \tag{13a}$$

$$B_\theta = \frac{\sin\theta}{r^3} \psi(r)\mu, \tag{13b}$$

where

$$f(r) = -\frac{3r^3}{r_g^3}\left[\ln\left(1-\frac{r_g}{r}\right) + \frac{r_g}{r} + \frac{1}{2}\left(\frac{r_g}{r}\right)^2\right], \tag{14}$$

$$\psi(r) = \frac{3r^2}{r_g^2}\left[\frac{1}{1-\frac{r_g}{r}} + 2\frac{r}{r_g}\ln\left(1-\frac{r_g}{r}\right) + 1\right]\sqrt{1-\frac{r_g}{r}}, \tag{15}$$

and μ is the magnetic dipole moment of a neutron star.

We perform a detail calculation of ρ_{GJ} with the magnetic field of a rotating neutron star given by equations (13a) and (13b), respectively. From equation (11), we find

$$\rho_{GJ} = -\frac{\Omega_0}{4\pi c r^2 \sin\theta}\left\{\left[\left(1-\frac{\kappa}{r^3}\right)\frac{r^3\sin^2\theta B_\theta}{\sqrt{1-r_g/r}}\right]_{,r} - \left[\left(1-\frac{\kappa}{r^3}\right)\frac{r^2\sin^2\theta B_r}{1-r_g/r}\right]_{,\theta}\right\}. \tag{16}$$

The calculation shows that

$$\rho_{GJ}(r,\theta) = \frac{3\Omega_0\mu}{4\pi c r^3}\left[F_1(\bar{r})\sin^2\theta - F_2(\bar{r})(\sin^2\theta - 2\cos^2\theta)\right], \tag{17}$$

with

$$F_1(\bar{r}) = \bar{r}^3\left\{((1-\frac{\beta}{\bar{r}^3})\left\{\frac{2}{\bar{r}-1} - \frac{1}{(\bar{r}-1)^2} + 2\ln(1-\frac{1}{\bar{r}})\right\}\right.$$
$$\left. +(2+\frac{\beta}{\bar{r}^3})\left\{\frac{1}{\bar{r}} + \frac{1}{\bar{r}-1} + 2\ln(1-\frac{1}{\bar{r}})\right\}\right\}, \tag{18}$$

$$F_2(r) = \bar{r}^3 \frac{2(1-\frac{\beta}{\bar{r}^3})}{1-\frac{1}{\bar{r}}} \left\{ \frac{1}{2\bar{r}^2} + \frac{1}{\bar{r}} + \ln(1-\frac{1}{\bar{r}}) \right\} . \tag{19}$$

Here, $\bar{r} = r/r_g$ and $\beta = \kappa/r_g^3$. Asymptotically as $r/r_g \to \infty$ functions $F_1(\bar{r})$ and $F_2(\bar{r}) \to 1$.

Thus Goldreich-Julian space charge has two purely general relativistic contributions, one is due to the Schwarzschild gravitoelectric parameter r_g and the second one is due to the gravitomagnetic Kerr parameter β. They have different dependence on r as $1/r$ and $1/r^3$, respectively. It is meant that near the surface of the star the gravitomagnetic term is in concurent with the gravitoelectric one. But in the distance from the surface of the star which is comparable with its radius R the gravitomagnetic term is ignorable small.

We plot $F_1(\bar{r})$ and $F_2(\bar{r})$ for $\beta = 0.1$. The dependence of these functions on $\bar{r}$ is shown in Fig.[1] and Fig[2], respectively. The Goldreich-Julian charge density under general relativity is shown in Fig.[3].

By least-squares fitting of the curve at $\theta = 0$, we find that Goldreich-Julian charge density decays with the distance away rom the star as follows:

$$\rho_{GJ} = \frac{10.5053}{r^3} - \frac{5.05692}{r^2} + \frac{1.06093}{r} - 0.084179 \ . \tag{20}$$

The charge density is maximum at the polar cap region and it remains almost same in certatin extented region in the pole. It is to be noted that the expression for ρ_{GJ} obtained by Muslimov and Harding (1997) shows the similar results.

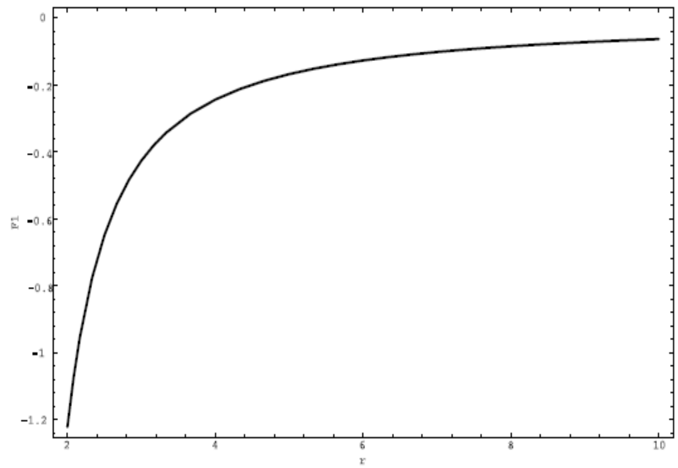

Fig.[1]. Goldreich-Julian charge density in general relativity; F_1 as function of $\bar{r}$.

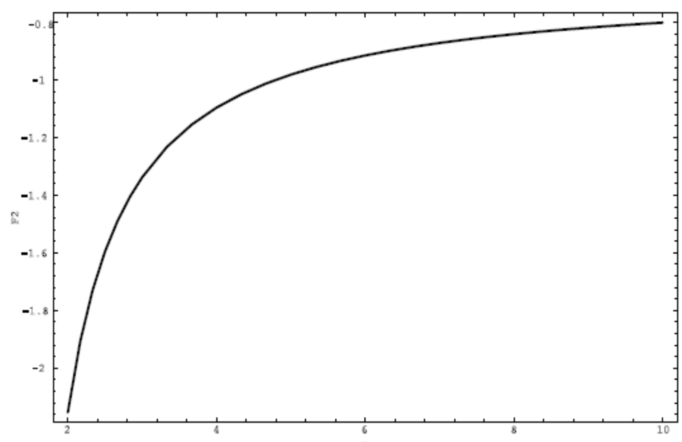

Fig.[2]. Goldreich-Julian charge density in general relativity; F_2 as a function of $\bar{r}$.

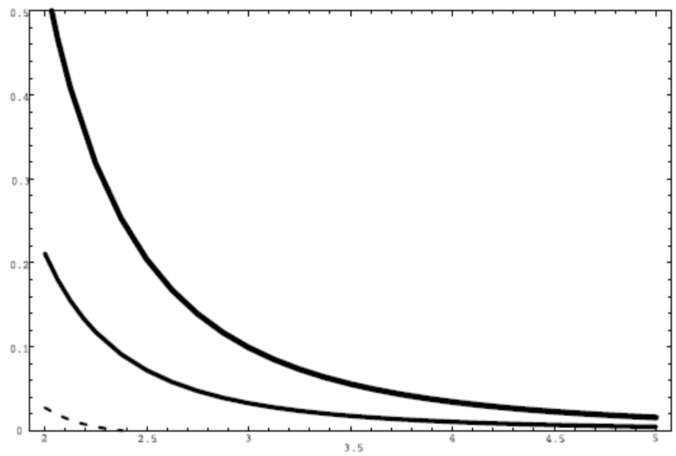

Fig.[3]. Goldrech-Julian charge density in general relativity ρ_{GJ} as function of $\bar{r}$. Thick line corresponds to $\theta = 0$, thin line to $\theta = \pi/4$, and broken line to $\theta = \pi/3$.

4. LINEAR PLASMA MODES ALONG THE OPEN FIELD LINES

The theory of cascade generation of electron-positron plasma at the polar cap region of a rotating plasma is developed by Ruderman & Sutherland (1975). According to the theoretical model, due to escape of charge particles along the open field lines , a polar potential gap is produced which continuously breaks down by forming electron positron pair on a time scale of a few microseconds. A photon of energy greater than $2mc^2$ produces an electron-positron pair. The electric field of the gap accelerates the positron out of the gap and accelerates the electron towards the stellar surface. The electron moves along a curved magnetic field line and radiate an energetic photon which goes on to produce a pair as it has a sufficient component of momentum perpendicular to the magnetic field. Recently, Zhy & Ruderman (1997) explained the $e - e^+$ pair production from a Crab-like pulsar. Electrons and positrons are accelerated in opposite directions to extremely high energies. The Lorentz factor γ of the primary electron and positron is given by

$$e\mathbf{E} \cdot \mathbf{B}c \approx \frac{e^2}{c^3}\gamma^4 \left(\frac{c^2}{r_c}\right)^2 ,$$

where This cascade of pair production, acceleration of electrons and positrons along curved field lines, curvature radiation-pair production results in a ”spark” break down of the gap.

Assuming a steady state thermodynamically equilibrium plasma state in the polar cap region, we study the linear plasma modes along the field lines. From the system of equations (10), (6) and (12), we derive the following linearized equations:

$$\nabla \cdot (\frac{1}{\sqrt{1-\frac{r_g}{r}}}\nabla\Phi) = -4\pi(\sum_s q_s \delta n_s - \rho_{GJ}) \tag{20}$$

$$\frac{\partial}{\partial t'}(\frac{\delta n_s}{n_0}) + \nabla \cdot (\sqrt{1-\frac{r_g}{r}}\mathbf{v}_s) = 0, \tag{21}$$

$$\frac{\partial \mathbf{v_s}}{\partial t'} = -\frac{q_s}{m}\nabla\Phi, \tag{22}$$

where $\partial/\partial t' = \partial/\partial t + \left(\frac{\kappa}{r^3} - 1\right)\Omega_0 \mathbf{m}\cdot\nabla$ is the global time derivative along ZAMO trajectories, $s(= e, e^+)$ is the plasma species, δn_s is the density fluctuation of the plasma species and n_0 is the equilibrium plasma density and ρ_{GJ} is the Goldreich-Julian charge density as defined by the equation (17). The system of equations (20)-(22) is equivalent to the following equation:

$$\frac{\partial}{\partial t'^2}\left(\nabla \cdot (\frac{1}{\sqrt{1-\frac{r_g}{r}}}\nabla\Phi) - 4\pi\rho_{GJ}\right) + \nabla \cdot (\omega_{p0}\sqrt{1-\frac{r_g}{r}}\nabla\Phi) = 0, \tag{23}$$

where $\omega_{p_0}^2 = 8\pi n_0 e^2/m$.

Now, by defining the electric field arising from charge separation and the corotational electric field, which is the source of ρ_{GJ}

$$\mathbf{E} \equiv -\frac{1}{\sqrt{1-\frac{r_g}{r}}}\nabla\Phi, \quad \rho_{GJ} = \frac{1}{4\pi}\nabla\cdot\mathbf{E}_c, \tag{24}$$

from equation (23), we find

$$\frac{\partial}{\partial t'^2}\left[\nabla\cdot(\mathbf{E}+\mathbf{E}_c)\right] + \nabla\cdot\left[\omega_{po}^2(1-\frac{r_g}{r})\mathbf{E}\right] = 0, \tag{25}$$

which gives

$$\frac{\partial}{\partial t'^2}\mathbf{E} + \omega_{po}^2\left(1 - \frac{r_g}{r}\right)\mathbf{E} = -\frac{\partial \mathbf{E}_c}{\partial t'^2}. \tag{26}$$

For $\frac{\partial \mathbf{E_c}}{\partial t'^2} = 0$ we may write the solution of equation (26) as

$$E = E_0 \exp\left\{-i\omega_{po}\sqrt{1 - \frac{r_g}{r}t'}\right\}. \tag{26}$$

From the above solution, we find that the plasma frequency in general relativity now is defined as

$$\omega_p^2 = \omega_{p0}^2\left(1 - \frac{r_g}{r}\right) \tag{28}$$

which is equivalent to the gravitational redshift of the oscillation. Fig.[4] shows the dependence of plasma oscillation on the distance away from the gravitational radius of the star.

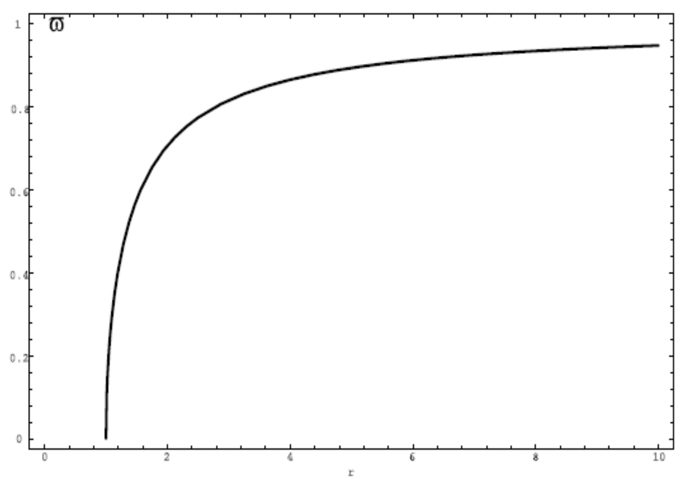

Fig.[4]. Plasma frequency $\omega_p(r)$ as a function of $\bar{r}$; gravitational redshift near the Schwarzschild radius.

The global time derivative along ZAMO trajectories is defined as

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \left(\frac{\kappa}{r^3} - 1\right)\Omega_0\frac{\partial}{\partial \phi}. \tag{29}$$

Thus, we may define

$$t' = t + \frac{\phi}{(\frac{\kappa}{r^3} - 1)\Omega_0}, \tag{30}$$

and hence the solution of linear plasma mode is

$$E(t,r,\phi) = E_0 \exp\left\{-i\omega_{p0}\sqrt{1-\frac{r_g}{r}}(t+\frac{\phi}{(\frac{\kappa}{r^3}-1)\Omega_0})\right\} \tag{31}$$

Introducing the dimensionless quantities $\epsilon = \frac{E}{E_0}, \tau = \omega_{po}t, x = \frac{r}{r_g}, \chi = \frac{\omega_{p0}}{\Omega_0}, \delta = \frac{\kappa}{{r_g}^3}$ from the equation (31), we find

$$\epsilon(\tau,x,\phi) = \sin\left[\sqrt{1-\frac{1}{x}}(\tau+\frac{\chi}{\frac{\delta}{x^3}-1}\phi)\right] \quad (32).$$

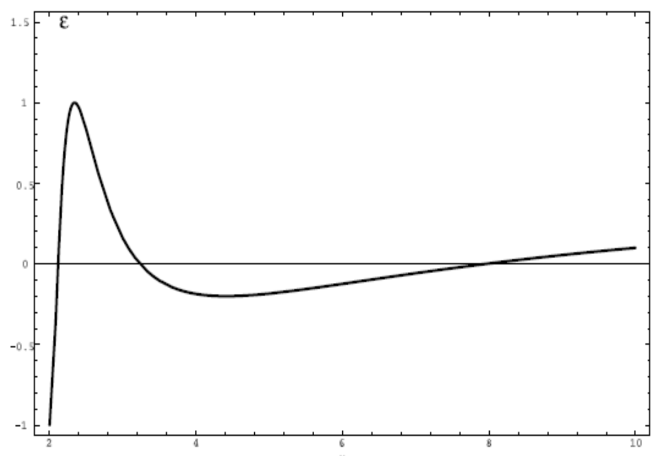

Fig.[5]. Linear plasma mode in general relativity; electrostatic field $\epsilon(\tau,\phi)$ as a function of $\bar{r}$.

We do some analysis of the linear electrostatic modes around a rotating neutron star. First, we consider the Schwarzschild radius equal to half the radius of the neutron star, i.e., $r_g = R/2$. Then we find $\delta = 8/5$. Considering relatively dense plasma, we put $\chi = 10^8$. We consider a fixed azimuthal angle $\phi = \pi/4$. For $\tau = 0$ we plot the field $\epsilon(\bar{r})$, which is shown in Figure 5. We find that the electrostatic field generated by Goldreich-Julian charge density is maximum near the star surface and falls quickly from the star.

5. NONLINEAR PLASMA MODES ALONG THE FIELD LINES

Now, we consider nonlinear plasma modes along the open field lines around a rotating neutron star. The system of equations governing the nonlinear modes can be written as

$$\frac{\partial}{\partial t'}(n_s) + \nabla \cdot (\sqrt{1-\frac{r_g}{r}} n_s \mathbf{v}_s) = 0, \tag{33}$$

$$\left(\frac{\partial}{\partial t'} + \sqrt{1-\frac{r_g}{r}} \mathbf{v_s} \cdot \nabla\right) \mathbf{v_s} = -\frac{q_s}{m} \nabla \Phi, \tag{34}$$

$$\nabla \cdot (\frac{1}{\sqrt{1-\frac{r_g}{r}}} \nabla \Phi) = -4\pi (\sum_s n_s q_s - \rho_{GJ}) \ . \tag{35}$$

For the simplicity, we consider $\mathbf{v_s}.\nabla \approx v_{sr} \cdot \nabla \partial/\partial r$ (i.e. one dimensional wave propagation along r) and introduce a moving frame $\eta = r - Vt'$, where V is a constant. In the considered moving frame, from equations (33) and (34), we get

$$n_s = \frac{n_0 V}{V - \sqrt{1-\frac{r_g}{\eta}} v_{s\eta}}, \tag{36}$$

$$v_{s\eta} = \frac{1}{mV}(q_s \Phi + \sqrt{1-\frac{r_g}{\eta}} \frac{e^2}{2mV^2} \Phi^2). \tag{37}$$

Using equations (36) and (37), in equation (35), we derive the nonlinear equation for the plasma mode along the field line of the rotating neutron star:

$$\frac{d^2\Phi}{d\eta^2} - \frac{r_g}{2\eta^2(1-\frac{r_g}{\eta})} \frac{d\Phi}{d\eta} + \frac{\omega_{po}^2(1-\frac{r_g}{r})}{V^2} \frac{\Phi}{1 - 2(1-\frac{r_g}{\eta})(\frac{e\Phi}{mV^2})^2 + \frac{1}{4}(1-\frac{r_g}{\eta})^2(\frac{e\Phi}{mV^2})^4}$$
$$= 4\pi \sqrt{1-\frac{r_g}{\eta}} \rho_{GJ} \ . \tag{38}$$

Now introducing dimensionless quantities

$$\Phi = \frac{e\Phi}{mv^2}, \qquad \bar{\eta} = \frac{\eta}{r_g}, \qquad \bar{\omega}_{p_0} = \frac{\omega_{p_0} r_g}{V} \tag{39}$$

we write the equation (38) in dimensionless form:

$$\frac{d^2\Phi}{d\bar{\eta}^2} - \frac{1}{2\eta(\bar{\eta}-1)}\frac{d\Phi}{d\bar{\eta}} + \bar{\omega}_{po}^2(1-\frac{1}{\bar{\eta}})\frac{\Phi}{1-2(1-\frac{1}{\bar{\eta}})\Phi^2+\frac{1}{4}(1-\frac{1}{\bar{\eta}})^2\Phi^4} = F_c. \tag{40}$$

where

$$F_c = -\frac{3\Omega_0\Omega_c r_g^2}{2V^2}\left(\frac{R}{\eta}\right)^3\sqrt{1-\frac{1}{\bar{\eta}}}\left[F_1(\bar{\eta})\sin^2\theta - F_2(\bar{\eta})(\sin^2\theta - 2\cos^2\theta)\right]. \tag{41}$$

Here $\Omega_c = eB_0/mc$; $F_1(\bar{\eta})$ and $F_2(\bar{\eta})$ are determined by the equations (18) and (19), respectively.

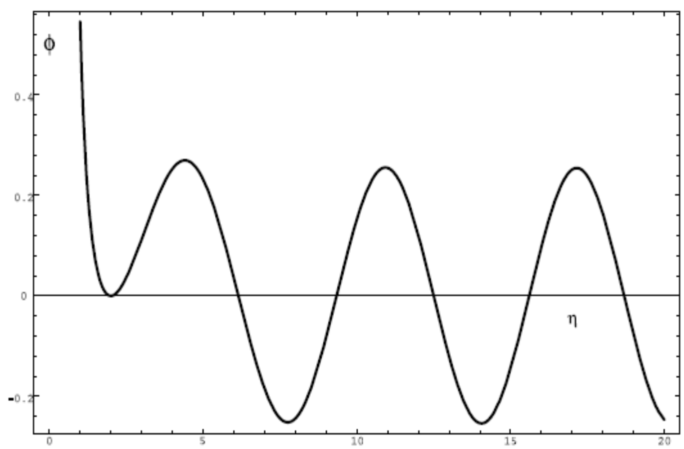

Fig.[6]. Nonlinear plasma mode in general relativity; propagation of plasma oscillation $\Phi(\bar{\eta})$ near the surface of a neutron star; numerical solution of the equation (40) with boundary condition $\Phi(2) = 0$, $\Phi'(2) = 0$.

We numerically solve the equation (40) in the polar cap region ($\theta \approx 0$) of a neutron star subject to the appropriate boundary conditions. Following Goldreich & Julian (1969) and Muslimov & Harding (1997), we assume that the surface of a polar cap and that formed by the last open field lines can be treated as electric equipotentials. We therefore adopt the condition $\Phi(r = R) = 0$. Second, we require that the steady state component of electric field parallel to magnetic field vanishes at the polar cap surface, i.e. $d\Phi(r = R)/d\eta = 0$. By considering $r_g = R/2$

for a neutron star, we write the boundary conditions as: $\Phi(2) = 0$ and $\Phi'(2) = 0$. The solution of the equation (40) with the mentioned boundary condition is shown graphically in Fig.[6]. We find that Goldrech-Julian charge density creates the initial potential on the surface. Near the radius the potential is enhanced and it propagates almost without damping along the field lines.

6. DISCUSSION AND CONCLUSION

We study the electrostatic plasma modes along the open field lines of a rotating neutron star. The dragging of inertial frame and the effect of general relativity is fully considered in this study. We perform a detailed analysis of Goldreich-Julian charge density in general relativity. Since pulsars having smaller obliquity have larger accelerating drops and this favored for γ -ray pulsar emissions (Muslimov 1995) and it supports the single pole γ - ray pulsar models (Daugherty & Harding 1994, 1996; Dermer & Sterner 1994), we confine our analysis in the zero inclination of the rotating neutron star. As pulsar radiation takes place in the plasma environment or the radiation passes through a plasma media , we consider the electrostatic plasma modes along the open field lines. We study both the linear and nonlinear modes in the neutron star or black hole plasma environment. Our general conclusion from the above analysis may be summarized as follows:

1. Goldreich-Julian charge density is maximum in the polar cap region and remains almost same in a certain extended region of the pole. The charge density exponentially decays with the distance away from the surface of the star.

2. Plasma oscillation along the field lines resembles to the gravitational redshift near Schwarzschild radius.

3. Plasma modes grows near the gravitational radius in the black hole environment.

4. For initially zero field on the surface of a rotating neutron star, Goldreich-Julian charge density, which is enhanced near the surface and propagates almost without damping along the open field lines, creates the plasma modes.

For further study of plasma dynamics in the neutron star or black environment we plan to extend our earlier investigations on solitons (see, Mofiz 1989, 1990, 1993 & Mofiz et al. 1985, 1995) propagation along the open field lines of strongly gravitating objects.

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